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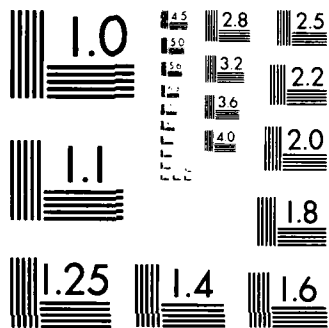
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LOW-ALTITUDE EARTH SATELLITE PROPELLANT LONGEVITY PREDICTION WITH APPLICATION TO FLIGHT PROFILE TRADE-OFF ANALYSIS

BY A. D. PARKS

STRATEGIC SYSTEMS DEPARTMENT

SEPTEMBER 1983

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FOREWORD

This study was conducted under the auspices of the Defense Mapping Agency in order to develop formal techniques that have important application to the solution of propellant management problems related to the selection of low-altitude earth satellite flight profiles. These techniques should help fill the analytical vacuum currently associated with this aspect of mission planning. This report was reviewed and approved by Dr. R. J. Anderle and Mr. R. W. Hill.

Released by:

A handwritten signature in dark ink, appearing to read 'T. A. Clare', is positioned above the printed name.

T. A. CLARE, Head
Strategic Systems Department

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INTRODUCTION

Several of the objectives to be satisfied by earth satellite mission planning analysis are to predict propellant lifetimes and to understand the various propellant conservation trade-offs that can be made available by adopting certain flight profiles. This aspect of mission planning is often of special importance for low-altitude missions, where considerable amounts of propellant are used for altitude maintenance thrusting to compensate for the effects of atmospheric drag deceleration.

Two methods are generally employed to perform propellant consumption studies:

1. The application of empirically derived propellant consumption rates
2. The utilization of analytic approaches based upon geopotential, drag, and discrete thrusting models of varying complexity

Both of these methods are quite effective. However, the first is often limited by the lack of data describing the effects of varying satellite and orbital geometries, as well as by solar and atmospheric conditions. The second approach often tends to be inefficient and tedious to use.

This paper presents an alternative (and perhaps more flexible) analytic development for predicting the rate at which propellant is consumed for in-track drag compensation thrusting for low-altitude earth satellites. The associated results can be effectively applied to propellant longevity analyses, especially to trade-off studies. The following sections discuss this development in detail, starting with derivations of the propellant longevity and mass decrement equations. These are then applied to spherically symmetric atmospheres that, under certain assumptions, can provide analytically tractable results. Such results are useful for comparison with those obtained from more complex cases under special limiting conditions. The more realistic and complex case of propellant consumption in an oblate diurnal atmosphere is also considered. An analytic expression for the associated mass decrement equation is derived and used in the following section to provide the reader with several illustrative numerical examples.

THE PROPELLANT LONGEVITY EQUATION

Consider an artificial earth satellite that has been inserted into an initial nominal orbit at time t_0 with a total weight of propellant ΔW_T available for orbit initialization and maintenance during its mission life. If the satellite were to operate in N different nominal orbits during its mission life, each requiring the weight of propellant ΔW_{0A_i} to achieve the i^{th} nominal orbit and using propellant at the rate \mathcal{R}_i during the interval $t_i - t_{i-1}$ while in the i^{th} nominal orbit, then the following expression may be written:

$$\Delta W_T + \sum_{i=1}^N \left\{ \int_{t_{i-1}}^{t_i} \mathcal{R}_i dt - \Delta W_{0A_i} \right\} = 0 \quad (1)$$

Note that ΔW_{0A_i} may result from orbital transfer thrusting, post-launch orbit initialization, and de-orbit thrusting.

Assume now that the propellant consumption rates \mathcal{R}_i can be separated into a radial thrusting rate \mathcal{R}_i^R , an in-track thrusting rate \mathcal{R}_i^I , a cross-track thrusting rate \mathcal{R}_i^C , and a miscellaneous operational maintenance consumption rate \mathcal{R}_i^{OM} ; then,

$$\mathcal{R}_i = \mathcal{R}_i^R + \mathcal{R}_i^I + \mathcal{R}_i^C + \mathcal{R}_i^{OM} \quad (2)$$

and Equation (1) becomes

$$\Delta W_T + \sum_{i=1}^N \int_{t_{i-1}}^{t_i} \left\{ \mathcal{R}_i^R + \mathcal{R}_i^C + \mathcal{R}_i^{OM} \right\} dt + \sum_{i=1}^{N'} \left\{ \int_{t_{i-1}}^{t_i} \mathcal{R}_i^I dt - \Delta W_{0A_i} \right\} = 0 \quad (3)$$

The prime on the second summation in the last equation denotes that no propellant is used if natural drag decay is used to establish shorter period orbits and that the summation should not include these natural decay intervals. The time bounds for such intervals are determined by the drag decay rates for those intervals.

This report is concerned only with the rate at which propellant is spent for in-track thrusting. Thus, the term of Equation (3) may be simplified by defining

$$\Delta W_{NIM} = - \sum_{i=1}^N \int_{t_{i-1}}^{t_i} \left\{ \mathcal{R}_i^R + \mathcal{R}_i^C + \mathcal{R}_i^{OM} \right\} dt \quad (4)$$

so that

$$\Delta W_T - \Delta W_{NIM} + \sum_{i=1}^{N'} \left\{ \int_{t_{i-1}}^{t_i} \mathcal{R}_i^I dt - \Delta W_{0A_i} \right\} = 0 \quad (5)$$

It is simpler to express Equation (5) in terms of the propellant mass decrement Δm (i.e., the in-track propellant mass expenditure for one orbital revolution). As will be shown in the following sections, this quantity is easily derivable for certain assumed conditions. This simplification is introduced through the following substitution:

$$\int_{t_{i-1}}^{t_i} \mathcal{R}_i^I dt \rightarrow g \sum_{j=1}^M \Delta m_{ij} \Delta n_{ij} \quad (6)$$

where the summation is taken over M groups of orbital revolutions in the i^{th} nominal orbit Δn_{ij} for which the mass decrement Δm_{ij} applies and g is the gravitational acceleration. Equation (5) then assumes the completely discreet form

$$\Delta W_T - \Delta W_{NIM} + \sum_{i=1}^{N'} \left\{ g \sum_{j=1}^M \Delta m_{ij} \Delta n_{ij} - \Delta W_{0A_i} \right\} = 0 \quad (7)$$

This equation shall be referred to in the following sections as the propellant longevity equation (PLE), since it can be used to compute the propellant life \mathcal{L} defined by

$$\mathcal{L} \equiv \sum_{i=1}^{N'} \sum_{j=1}^M \Delta n_{ij} \tau_{ij} + \Delta \mathcal{L}_D \quad (8)$$

where τ_{ij} is a nominal orbit period for Δn_{ij} orbital revolutions and $\Delta \mathcal{L}_D$ is the total time spent during natural decay phases.

THE MASS DECREMENT EQUATION

Assume that an artificial earth satellite in a nominal orbit with semimajor axis a and eccentricity e is continuously experiencing atmospheric drag deceleration and is simultaneously performing in-track microthrusting to offset the effects of drag decay so that the nominal

orbit is maintained. Let dE_D be an infinitesimal orbital energy change induced by drag decay and dE_T be an infinitesimal energy change produced by the microthrusting. In order that the nominal orbit be maintained, the following condition must be satisfied:

$$dE_T = - dE_D \quad (9)$$

The work done by the drag force (which is assumed to operate only in the direction opposite the satellite velocity vector \vec{v}) in the infinitesimal time element dt is

$$dE_D = - \frac{\delta}{2} C_D A \rho |\vec{v}|^3 dt \quad (10)$$

where C_D is the drag coefficient, A is the cross sectional area of the satellite in the direction of motion, and ρ is the atmospheric density at the satellite's location. The δ factor accounts for the orbital orientation with respect to a rotating atmosphere and has the form

$$\delta = \left[1 - \left(\frac{r_p}{v_p} \right) \Lambda \omega_e \cos i \right]^2 \quad (11)$$

where r_p and v_p are the radius and velocity of the satellite at perigee, respectively, ω_e is the earth's angular rotation rate, i is the orbital inclination, and Λ is the ratio of the atmospheric to earth angular rotation rates. The work done by in-track microthrusting in the same infinitesimal time element dt is

$$dE_T = T |\vec{v}| dt \quad (12)$$

or, after applying the "rocket equation",

$$dE_T = - g I_{sp} |\vec{v}| dm \quad (13)$$

where T is the thrust, I_{sp} is the thruster specific impulse, and dm is the infinitesimal element of mass of the propellant expended during the thrust interval dt .

Use of Equations (10) and (13) in Equation (9) allows the in-track propellant expenditure rate equation for drag deceleration offset to be written in terms of the satellite's orbital and drag characteristics, as well as atmospheric and thruster properties:

$$\frac{dm}{dt} = - \frac{C_D A \delta}{2g I_{sp}} |\vec{v}|^2 \rho \quad (14)$$

The in-track mass expenditure during one orbital revolution (i.e., the mass decrement) is obtained by integrating the last equation over one orbital period:

$$\Delta m = - \frac{C_D A \delta}{2g I_{sp}} \int_0^T |\vec{v}|^2 \rho dt \quad (15)$$

It is convenient to change the independent variable from t to E , the eccentric anomaly. This is done using the following relationships:

$$|\vec{v}|^2 = \left(\frac{\mu}{a} \right) \left(\frac{1 + e \cos E}{1 - e \cos E} \right) \quad (16)$$

and

$$dt = \left(\frac{a^3}{\mu} \right)^{1/2} (1 - e \cos E) dE \quad (17)$$

where μ is the earth's gravitational constant. Using Equations (16) and (17) in Equation (15) and making the upper integration bound consistent with the new independent variable gives

$$\Delta m = - \frac{C_D A \delta}{2g I_{sp}} (\mu a)^{1/2} \int_0^{2\pi} (1 + e \cos E) \rho dE \quad (18)$$

In forming this expression, it is assumed that C_D , A , and I_{sp} are constant over the revolution. This equation will be referred to hereafter as the mass decrement equation (MDE).

PROPELLANT CONSUMPTION IN SPHERICALLY SYMMETRICAL ATMOSPHERES

The following subsections are devoted to the consideration of the cases where the atmospheric density is assumed to be spherically symmetrical. These cases are of interest because of their analytic tractability. Although their utility may be somewhat limited, they can provide interesting insights into the more general problem.

CONSTANT DENSITY

This subsection is concerned with the case where the atmospheric density is assumed to be spherically symmetrical and constant with altitude above the surface of the earth. If

$$\rho = \rho_o \quad (19)$$

where ρ_o is a constant, then Equation (18) becomes

$$\Delta m = - \frac{C_D A \delta}{2g I_{sp}} (\mu a)^{1/2} \rho_o \int_0^{2\pi} (1 + e \cos E) dE \quad (20)$$

or

$$\Delta m = - \frac{\pi C_D A \delta}{g I_{sp}} (\mu a)^{1/2} \rho_o \quad (21)$$

This result may be readily applied to the PLE to give

$$\Delta W_T - \Delta W_{NIM} + \sum_{i=1}^{N'} \left\{ - \frac{\pi C_D A \delta}{I_{sp}} (\mu a_i)^{1/2} \rho_o \sum_{j=1}^M \Delta n_{ij} - \Delta W_{0A_i} \right\} = 0 \quad (22)$$

Further simplification may be introduced by restricting this application to orbits having the same nominal orbital period (i.e., the same a), but different eccentricities. Since

$$n_i = \sum_{j=1}^M \Delta n_{ij} \quad (23)$$

and

$$n' = \sum_{i=1}^{N'} n_i \quad (24)$$

where n' is the total number of orbital revolutions spent in the N nominal orbits (excluding those occurring during natural decay phases), then it is found that

$$n' = \frac{\left[\Delta W_T - \Delta W_{NIM} - \sum_{i=1}^{N'} \Delta W_{0A_i} \right] I_{sp}}{\pi C_D A \delta (\mu a)^{1/2} \rho_o} \quad (25)$$

The associated propellant life \mathcal{L} is obtained by using Equations (23) through (25) and the relation

$$\tau = 2\pi \left(\frac{a^3}{\mu} \right)^{1/2} \quad (26)$$

so that

$$\mathcal{L} = \frac{2 \left[\Delta W_T - \Delta W_{NIM} - \sum_{i=1}^{N'} \Delta W_{0A_i} \right] a I_{sp}}{C_D A \delta \mu \rho_o} + \Delta \mathcal{L}_D \quad (27)$$

This result substantiates what one expects intuitively about propellant longevity, i.e., propellant life is enhanced when

1. Less propellant is used for operational maintenance and configuration change orbit adjusts
2. The nominal semimajor axis is increased
3. The thruster specific impulse is increased
4. The drag coefficient and satellite cross sectional area are decreased
5. The atmospheric density is decreased

EXPONENTIALLY DECREASING DENSITY

In this section, an atmospheric density model is used that assumes that the density depends solely upon the distance r from the center of the earth and varies exponentially with r . This density model has the form

$$\rho = \rho_o \exp(-\beta(r - r_o)) \quad (28)$$

where r_{p_0} is the perigee distance from the earth's center, ρ_{p_0} is the density at the perigee point, and

$$\beta = \frac{1}{H} \quad (29)$$

where H is the density scale height and is assumed constant. Substituting the relations

$$r = a(1 + e \cos E) \quad (30)$$

and

$$r_{p_0} = a(1 - e) \quad (31)$$

into Equation (28) gives

$$\rho = \rho_{p_0} \exp [-\beta a e + \beta a e \cos E] \quad (32)$$

Using this expression in the MDE gives

$$\Delta m = \frac{C_D A \delta}{2g I_{sp}} (\mu a)^{-1} \rho_{p_0} \exp [-\beta a e] \int_0^{2\pi} (1 + e \cos E) \exp [\beta a e \cos E] dE \quad (33)$$

$$\Delta m = \frac{\pi C_D A \delta}{g I_{sp}} (\mu a)^{-1} \rho_{p_0} \exp [-\beta a e] \left\{ I_0(\beta a e) + e I_1(\beta a e) \right\} \quad (34)$$

where $I_n(x)$ is Bessel function of the first kind and imaginary argument defined as

$$I_n(x) = \frac{1}{\pi} \int_0^\pi [x \cos(\theta)]^n \cos(n\theta) d\theta \quad (35)$$

Equation (34) can be substituted into the PLE to give

$$\Delta W_F - \Delta W_{NIM} + \sum_{i=1}^{N'} \left\{ - \frac{\pi C_D A \delta}{I_{sp}} (\mu a_i)^{1/2} \rho_{p_{o_i}} \exp[-\beta_i a_i e_i] \left(\frac{1}{2} I_0(\beta_i a_i e_i) + e_i I_1(\beta_i a_i e_i) \right) + \sum_{j=1}^M \Delta n_{ij} - \Delta W_{oA_i} \right\} = 0 \quad (36)$$

Further simplification of this expression is difficult except for the case where $N = 1$. Then, using Equation (24), one finds that

$$R' = \frac{[\Delta W_F - \Delta W_{NIM} - \Delta W_{oA}] I_{sp} \exp[\beta a e]}{\pi C_D A \delta (\mu a)^{1/2} \rho_{p_o} [I_0(\beta a e) + e I_1(\beta a e)]} \quad (37)$$

so that the PLE becomes

$$\mathcal{L} = \frac{2[\Delta W_F - \Delta W_{NIM} - \Delta W_{oA}] a I_{sp} \exp[\beta a e]}{C_D A \delta \mu \rho_{p_o} [I_0(\beta a e) + e I_1(\beta a e)]} + \Delta \mathcal{L}_D \quad (38)$$

This expression also substantiates the five points concerning propellant life enhancement that were discussed earlier. It should be noted that Equation (38) reduces to the result of Equation (27) for $N = 1$ when the eccentricity is zero, since

$$I_0(0) = 1 \quad (39)$$

PROPELLANT CONSUMPTION IN AN OBLATE ATMOSPHERE WITH DAY-TO-NIGHT DENSITY VARIATION

In this section, an analytic expression is developed for the MDF for a low-altitude satellite operating in an orbit with small eccentricity ($0.01 \lesssim e \lesssim 0.10$). The form of the atmospheric density model is that of an oblate atmosphere with day-to-night density variation.

An analytic form for the atmospheric density can be obtained by combining the oblate atmosphere form used by Cook, King-Hele, and Walker¹ with that of an atmosphere with diurnal variation discussed by Cook and King-Hele². The resulting form for the density is given by

$$\rho = \rho_o (1 + F \cos \phi) \exp [-\beta(r - \sigma)] \quad (40)$$

where ρ_o is a reference atmospheric density of the form

$$\rho_o = \frac{1}{2} (\rho_{\max} + \rho_{\min}) \quad (41)$$

$$F = \frac{\rho_{\max} - \rho_{\min}}{\rho_{\max} + \rho_{\min}} \quad (42)$$

$$\beta = H^{-1} \quad (43)$$

$$\sigma = r_p \left\{ \frac{1 - \epsilon \sin^2 i \sin^2 u}{1 - \epsilon \sin^2 i \sin^2 \omega} \right\} \quad (44)$$

and

$$\cos \phi = A \left[\frac{\cos E - e}{1 - e \cos E} \right] + B \left[\frac{(1 - e^2)^{1/2} \sin E}{1 - e \cos E} \right] \quad (45)$$

In the last four equations, ρ_{\max} and ρ_{\min} are the maximum day time and minimum night time densities, respectively; H is the density scale height; i and ω are the orbital inclination and argument of perigee, respectively; u ($= \omega + \theta$, where θ is the true anomaly) and E are the true argument of latitude and eccentric anomaly, respectively; ϵ is the earth's ellipticity; r_p is the perigee radius; and

$$A = \sin \delta_B \sin i \sin \omega + \cos \delta_B \left\{ \cos(\Omega - \alpha_B) \cos \omega - \cos i \sin(\Omega - \alpha_B) \sin \omega \right\} \quad (46)$$

and

$$B = \sin \delta_B \sin i \cos \omega - \cos \delta_B \left\{ \cos(\Omega - \alpha_B) \sin \omega + \cos i \sin(\Omega - \alpha_B) \cos \omega \right\} \quad (47)$$

¹G. E. Cook, D. G. King-Hele, and D. M. C. Walker, *Proceedings of the Royal Society A*, **264**, pp. 88-121, 1961.

²G. E. Cook, and D. G. King-Hele, *Proceedings of the Royal Society A*, **259**, pp. 33-67, 1965.

where α_B and δ_B are the right ascension and declination of the atmospheric diurnal bulge, respectively, and Ω is the right ascension of the ascending node of the satellite orbit.

Equation (44) may be expanded to first-order in ϵ to give

$$\sigma = r_p \left[1 + \frac{1}{2} \epsilon \sin^2 i (\cos 2u - \cos 2\omega) \right] \quad (48)$$

Similarly, Equation (45) may be expanded to first-order in e to give

$$\cos \phi = A(\cos E - e + e \cos^2 E) + B(\sin E + e \sin E \cos E) \quad (49)$$

Substituting Equations (48) and (49) into Equation (40) and using the relation

$$r = a(1 - e \cos E) \quad (50)$$

allows the following first-order expression to be written for the atmospheric density:

$$\rho = \rho_o \left[1 + FA(\cos E - e + e \cos^2 E) + FB(\sin E + e \sin E \cos E) \right] \quad (51)$$

$$\exp \left\{ -\beta a e (1 - \cos E) + e \cos 2u - e \cos 2\omega \right\}$$

where

$$e = \frac{1}{2} \epsilon \beta r_p \sin^2 i \quad (52)$$

As discussed in Reference 1, e may be treated as a small parameter of the same order of magnitude as the eccentricity. Thus, in Equation (51), the following expansion may be used:

$$\exp \left\{ e \cos 2u \right\} = 1 + e \cos 2u + \frac{1}{2} e^2 \cos^2 2u \quad (53)$$

Substituting Equations (51) and (53) into Equation (18) and using the relations

$$\cos \theta = \frac{\cos E - e}{1 - e \cos E} \quad (54)$$

and

$$\sin \theta = \frac{(1 - e^2)^{1/2} \sin E}{1 - e \cos E} \quad (55)$$

to eliminate θ to first order in e gives

$$\begin{aligned} \Delta m = & - \frac{C_D}{2g} \frac{A}{I_{sp}} (\mu a)^{1/2} \rho_o \exp \{ - \beta a e - c \cos 2\omega \} \int_0^{2\pi} \{ [1 + FA(\cos E - e + e \cos^2 E) \\ & + FB(\sin E + e \sin E \cos E)] \cdot [1 + c \cos 2(\omega + E) - 2ec \sin 2(\omega + E) \sin E \\ & + \frac{1}{4} c^2 + \frac{1}{4} c^2 \cos 4(\omega + E) - ec^2 \sin 4(\omega + E) \sin E] \cdot \\ & [1 + e \cos E] \exp [\beta a e \cos E] \} dE \end{aligned} \quad (56)$$

When the integrand of this equation is multiplied, the result contains trigonometric terms that are expressible as functions of $\cos(nE)$, $n = 0, 1, 2, \dots, 6$. This allows Equation (56) to be written in terms of the integral representation of the Bessel function of the first kind and imaginary argument defined by

$$I_n(\beta a e) = \frac{1}{2\pi} \int_0^{2\pi} \cos(nE) \exp(\beta a e \cos E) dE \quad (57)$$

The resulting MDF for an oblate diurnal atmosphere is

$$\begin{aligned}
\Delta m = & - \frac{\pi C_D A \delta}{g I_{sp}} (\mu a)^{1/2} \rho_o \exp \left\{ -\beta a e - c \cos 2 \omega \right\} \left[\left(1 + \frac{c^2}{4} \right) (I_o + e I_1) + \right. \\
& \text{FA} \left[\left(1 + \frac{c^2}{4} \right) (I_1 + e I_2) \right] + \frac{c}{2} \left\{ [2I_2 - e(I_1 - 3I_3)] + \text{FA} [I_1 + I_3 + \right. \\
& \left. 2eI_4] \right\} \cos 2 \omega - \frac{c}{2} \text{FB} \left\{ (I_1 - I_3) + 2e (I_2 - I_4) \right\} \sin 2 \omega + \\
& \frac{c^2}{8} \left\{ [2I_4 - e(3I_3 - 5I_5)] + \text{FA} [(I_3 + I_5) + e (3I_6 - I_2)] \right\} \cos 4 \omega + \\
& \left. \frac{c^2}{8} \text{FB} \left\{ [I_5 - I_3] + e[I_2 - 4I_4 + 3I_6] \right\} \sin 4 \omega \right] \quad (58)
\end{aligned}$$

where the Bessel function argument $\beta a e$ has been suppressed for the sake of brevity.

This MDE could be introduced into the PLE at this point. However, due to its complexity and dependence upon solar position and the additional orbital parameters Ω and ω , little, if any, simplification could be introduced into the resulting analytic expression. It should be noted, nonetheless, that Equation (58) reduces to the form of Equation (34) when oblateness and diurnal effects are neglected; i.e., when

$$A \rightarrow 0$$

$$B \rightarrow 0$$

and

$$c \rightarrow 0$$

NUMERICAL EXAMPLES

Several numerical examples that illustrate the types of analyses to which the MDE for an oblate diurnal atmosphere may be applied are presented in this section. These examples were created using analytically modeled averaged variational equations³ to represent the effects

³ L. J. L. Liu and R. L. Alford, "Semianalytic Theory for a Close-Earth Satellite," *AIAA Journal of Guidance and Control*, Vol. 3, July-August 1980, pp. 304-311.

of geopotential perturbations on the satellite motion through J_4 . Theoretical expressions for drag decay rates for a low-altitude satellite orbiting in an oblate diurnal atmosphere⁴ were used to model the natural drag decay phase occurring between the time of propellant depletion and reentry. The MDE of Equation (58) was used to compute the propellant requirements for orbit sustenance. Changes in orbit parameters occurring during orbit adjusts were computed using results obtained from the Lagrange planetary equations when impulsive velocity changes are assumed.⁵ The quantities of propellant expended during an orbit adjust were obtained from application of the "rocket equation."

The Jacchia 1960 model atmosphere was used for all density computations required for both the MDE and the drag decay rates: i.e., computation of the density ρ_{max} at the diurnal bulge location and the density ρ_{min} diametrically opposite the bulge at the satellite's osculating perigee altitude. This model atmosphere describes the density variation with altitude of an oblate diurnal atmosphere and accounts for the effects of density variation due to solar activity via a dependence upon the solar flux $F_{10.7}$. It is not believed to be extremely representative of the atmospheric density, but is computationally very efficient. Santora's method for density scale height selection⁴ was employed for both drag decay rate and MDE evaluation.

The following data were used to initialize the computations for each of the examples below:

$$\begin{aligned}
 \bar{a} &= 6756.205 \text{ km} \\
 \bar{e} &= 0.009656113 \\
 \bar{i} &= 94.99996^\circ \\
 \bar{\omega} &= 193.3874^\circ \\
 \bar{\Omega} &= 95.00015^\circ \\
 \bar{M} &= 156.6115^\circ \\
 t_o &= 44619.987 \text{ Modified Julian Days} \\
 I_{sp} &= 230 \text{ sec} \\
 \Delta W_T &= 150 \text{ kg}
 \end{aligned} \tag{59}$$

AVERAGE IN-TRACK CONSUMPTION RATE PREDICTION

Computations were performed using the conditions of Equation (59) for $F_{10.7}$ values between 100 and 300 flux units. Three $C_D A$ values of $2. \times 10^{-4} \text{ km}^2$, $1. \times 10^{-4} \text{ km}^2$, and $5. \times 10^{-5} \text{ km}^2$ were assumed. The average in-track consumption rate $\bar{\dot{R}}^1$ was obtained

⁴ E. A. Santora, "Satellite Drag Perturbations in an Oblate Diurnal Atmosphere," *ALAA Journal*, Vol. 13, September 1975, pp. 1212-1216.

⁵ A. D. Fays, *Instantaneous Orbit Parameter Changes Produced by Impulsive Thrusting with Application to Orbit Adjust Design for Satellites on Small Eccentricity Orbits*, NSWC/DL TR 83-31 (Dahlgren, Va., February 1983), pp 3-4.

for each $F_{10.7}$ and $C_D A$ combination by dividing the 150 kg of propellant consumed by the time required for its consumption (i.e., the length of time the orbit was sustained). The results are presented in Figure 1. It should be noted that those rates are representative only of those required for sustenance of the orbit described by the elements of Equation (59), since prereentry drag decay phases are not included.

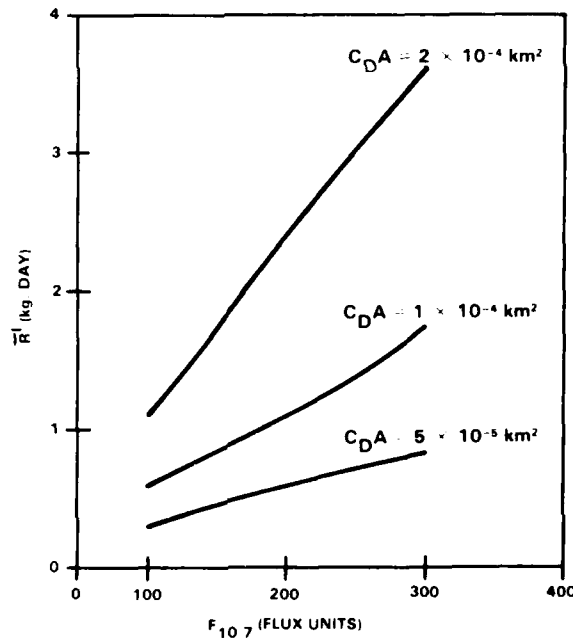


FIGURE 1. AVERAGE IN-TRACK PROPELLANT CONSUMPTION RATES \bar{R}^t AS A FUNCTION OF SOLAR FLUX CONDITIONS AND SATELLITE DRAG CHARACTERISTICS

SAMPLE FLIGHT PROFILE TRADE-OFF ANALYSIS

Consider the following scenario: "A satellite mission is ordinarily constrained to operate in an orbit described by the parameters of Equation (59) until its propellant is depleted, at which time it deorbits by natural drag decay. After orbital insertion, it becomes apparent that it will likely be necessary to take measures at sometime during the mission to extend the mission length for as long as possible." The MDE can be applied to the situation depicted by this scenario to provide estimates of the propellant and orbital lifetime trade-offs that exist.

Results are presented for the special case where the semimajor axis is increased 30 km by an in-track apogee thrust at some time during the mission and the satellite deorbits by natural drag decay. A representative flight profile is presented in Figure 2. The desired data were generated using $F_{10} = 100$ and $C_D A = 1. \times 10^{-4} \text{ km}^2$ and are shown in Figure 3. There the change in the propellant longevity and the increase in total mission life are plotted against the revolution number during which the orbit adjust was performed. For example, if the adjust was performed on rev 420, there would be no change in the propellant longevity over that if no adjust was performed. The mission life would also be extended by 190 days due to the decrease in the drag decay rate obtained by operating at a higher altitude.

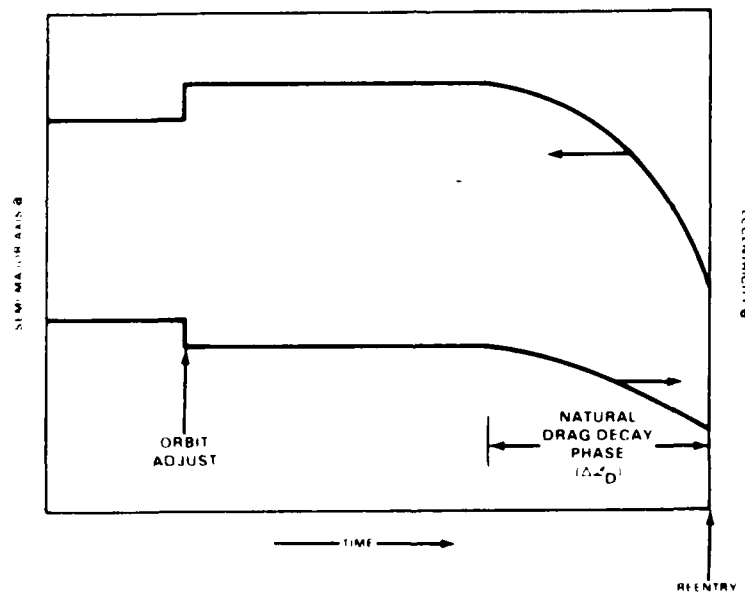


FIGURE 2. REPRESENTATIVE FLIGHT PROFILE

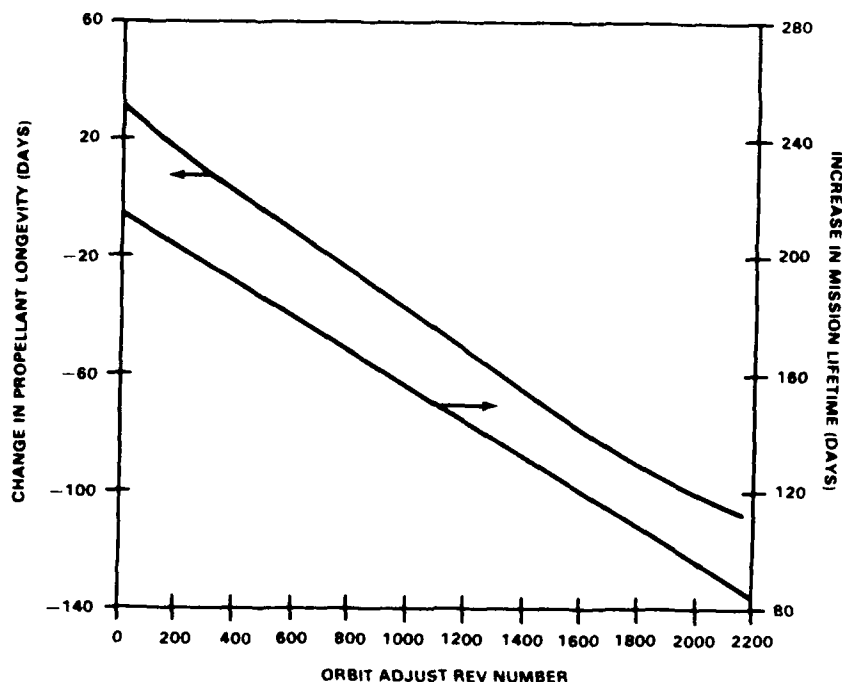


FIGURE 3. IMPACT OF ORBIT MODIFICATION UPON PROPELLANT AND MISSION LIFETIMES

SUMMARY

The propellant longevity equation has been derived in both its continuous and discrete forms and expressions for the mass decrement equation in spherically symmetrical and oblate diurnal atmospheres have been developed. The mass decrement equation for an oblate diurnal atmosphere has been applied to the discrete form of the propellant longevity equation to provide several numerical examples that illustrate their applicability to the solution of certain mission planning problems.

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